

Practice problems 1

1. A bank has the following data on the gender and marital status of 200 customers.

	Male	Female
Single	25	25
Married	100	50

- What is the probability of finding a single female customer?
 - What is the probability of finding a male customer or a married customer?
 - If a customer is male, what is the probability that he is married?
2. If A and B are independent events with $P(A) = 0.05$ and $P(B) = 0.65$, then $P(A|B) =$
3. In a Calculus class there are 11 freshmen and 15 sophomores; 5 of the sophomores are females, and 8 of the freshmen are males. If a student is selected at random, what is the probability of selecting a sophomore or a male?
4. The New England Life Insurance Company issues one-year policies to 12 men who are all 27 years of age. Based on data from the Department of Health and Human Services, each of these men has a 99.82% chance of living through the year. What is the probability that they all survive the year?
5. An executive on a business trip must rent a car in each of two different cities. Let A denote the event that the executive is offered a free upgrade in the first city and B represent the analogous event for the second city. Suppose that $P(A) = .3$, $P(B) = .4$, and that A and B are independent events.
- If the executive is not offered a free upgrade in the first city, what is the probability of not getting a free upgrade in the second city? Explain your reasoning.
 - What is the probability that the executive is offered a free upgrade in at least one of the two cities?
 - If the executive is offered a free upgrade in at least one of the two cities, what is the probability that such an offer was made only in the first city?
6. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared.
- If it has an emergency locator, what is the probability that it will not be discovered?
 - If it does not have an emergency locator, what is the probability that it will be discovered?

7. Determine the constant c so that the following represents a probability distribution, then compute its expected value, variance and standard deviation.

$$P(x) = c|x - 3|, x = 0, 1, 2, 3, 4, 5.$$

8. A package of 8 batteries is checked to determine if there are any dead batteries. Four batteries are checked. If one or more are dead, the package is not sold. What is the probability that the package will not be sold, if there are actually two dead batteries in the package?
9. The failure rate for taking the bar exam in Central City is 35%. If 250 people take the bar exam, what are the mean, variance and standard deviation for the number of failures?
10. An estimated 2.3 million people are poisoned each year by dangerous chemicals and products found in the home. 64% involve children under the age of 6. Out of the next four calls to the poison control center, what is the probability at least two are for children under the age of 6?
11. A roofing company receives on the average 4 calls per day. On any given day, what is the probability that it will receive at most 2 calls? What is the probability that it will receive at least 4 calls?
12. In a 600-page manuscript there are 150 randomly selected misprints. If a page is selected at random, what is the probability that it had 2 misprints?
13. An industrial company carries out its operations 50 weeks per year. Accidental injuries occur at the company according to a Poisson process with the mean rate of three per week.
- Find the probability of more than one accidental injury during a given week.
 - Find the probability that in a given year there will be at least one accidental injury during every week of operation.
14. An archer shoots at a circular target. Suppose the distance between the center and the impact point of the arrow is a continuous random variable X (in meters) with the probability density function given by
- $$f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
- Find the cumulative distribution function (cdf) $F(x)$ for all values of x .
 - The bull's-eye of the target is a circle of radius 10 cm. What is the probability that the archer hits the bull's-eye?

15. The proportion of people who respond to a certain mail-order solicitation is a random variable X having density function given by

$$f(x) = \begin{cases} kx(x+2), & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

- a) Find the constant k .
b) Find $E(X)$ and $V(X)$.

16. The assembly time for a product is uniformly distributed between 6 to 10 minutes.

- (a) Find the probability of assembling the product between 7 and 9 minutes.
(b) Find the expected assembly time.

17. Using the standard normal distribution, find $P(-1.28 < Z < 0.2)$, and $P(Z > -1.64)$. Also, find z such that $P(Z \leq z) = 0.9$

18. The weight of football players is normally distributed with a mean of 200 lbs and a standard deviation of 25 lbs.

- a) Find the probability of a player weighing less than 250 lbs.
b) What percent of players weigh between 180 and 220 lbs?
c) What is the minimum weight of the middle 95% of the players?

19. Suppose that X and Y have a discrete joint distribution defined by:

$$f(x, y) = \begin{cases} c |x + y| & \text{for } x = -2, -1, 0, 1, 2 \text{ and } y = -2, -1, 0, 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

- a) Determine the value of c .
b) Find $P(X = 0, Y = -2)$, $P(X = 1)$ and $P(|X - Y| \leq 1)$.
c) Find the marginal distributions of X and Y .
d) Determine whether X and Y are independent. Justify your answer.

20. Vehicles arrive at a highway booth at random instances from both the south and north. Assume that they arrive at average rates of five and three per 5-minute period, respectively. Let X denote the number arriving from the south during a 5-minute period, and let Y denote the number arriving from the north during this same time. Assume that X and Y are independent.

- a) Find the joint probability distribution of (X, Y) .
b) Find the probability that a total of four vehicles arrives during a five minute period.
c) Find the total number of vehicles expected to arrive at the booth from both directions during one hour.

21. If the moment-generating function of X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, find the mean, variance and p.m.f. of X .

- 22.** The length of time for an individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on a given day?