Homework #2 selected answers

Section 2.2: Ex. 35. a) You can write the system obtained from solving $Z = aX + bY$, or by inspection we observe that $Z = 3X - Y$.
b) Write the augmented matrix for the system obtained from solving for a and b such that $W = aX + bY$.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

But, since the last row in the augmented matrix is [0 0 1], this corresponds to an equation 0=1 which is impossible, so the system is inconsistent.
c) Follows from the above since the matrix of the homogeneous system is now the upper triangular matrix obtained above. That implies that c=b=a=0, so the only solution to the system is the trivial one.
Since we saw in part a) that $Z = 3X - Y$, we have $3X - Y - Z = 0$, so we obtained a linear combination of the vector matrices X,Y and Z equal to zero, but with coefficients not all equal to zero.

Ex. 44. $A = \begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}, A^2 = \begin{bmatrix} 56 & -40 \\ 20 & -4 \end{bmatrix}$.

Then, if $f(x) = x^2 - 10x + 24, f(A) = A^2 - 10A + 24I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Ex. 60. Let A and B be skew-symmetric matrices. This means that $A = -A^T, B = -B^T$.
Then $(A+B)^T = A^T + B^T = -A + (-B) = -(A+B)$ showing that A+B is skew-symmetric.

Section 2.3: Ex 26(a) The coefficient matrix is $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$, so $A^{-1} = \frac{1}{5}\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

Then $X = \begin{bmatrix} x \\ y \end{bmatrix} = A^{-1}\begin{bmatrix} 5 \\ 10 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -10 \\ 15 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

Ex. 36. For the matrix to be singular we need to have $4(-3) + 2x = 0, x = 6$.
Ex. 38. We saw that for matrix of order 2 to be invertible we must have non proportional rows, therefore $\sin^2\theta + \cos^2\theta \neq 0$ which is always true since by the Pythagorean theorem $\sin^2\theta + \cos^2\theta = 1$.

Ex. 46. Since $ABC = I = A(BC)$, it follows that A has an inverse and $A^{-1} = BC$. Similarly, $C^{-1} = AB$. Then we have: $B = A^{-1}(ABC)C^{-1} = A^{-1}IC^{-1} = A^{-1}C^{-1}$. Then, $B^{-1} = (A^{-1}C^{-1})^{-1} = CA$.