Math 3260

Homework #4

Ex. 6 section 3.4.

The characteristic equation is $\begin{vmatrix} 2-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} = 0 \Leftrightarrow (2-\lambda)^2 - 4 = 0, 2-\lambda = \pm 2$, so we get $\lambda_1 = 0, \lambda_2 = 4$.

For $\lambda_1 = 0$ we get the eigenvectors by solving the homogeneous linear system whose coefficient matrix is $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$, so the eigenvectors are $(t, -t), t \in R$.

For $\lambda_1 = 4$ we get the eigenvectors by solving the homogeneous linear system whose coefficient matrix is $\begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}$, so the eigenvectors are $(t, t), t \in R$.

Ex. 8 section 3.4.

The characteristic equation is $\begin{vmatrix} 2-\lambda & 5 \\ 4 & 3-\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - 5\lambda - 14 = 0, \lambda_1 = 7, \lambda_2 = -2$.

For $\lambda_1 = 7$ we get the eigenvectors by solving the homogeneous linear system whose coefficient matrix is $\begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, so the eigenvectors are $(t, t), t \in R$.

For $\lambda_1 = -2$ we get the eigenvectors by solving the homogeneous linear system whose coefficient matrix is $\begin{bmatrix} 4 & 5 \\ 4 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 5/4 \\ 0 & 0 \end{bmatrix}$, so the eigenvectors are $(-\frac{5t}{4}, t), t \in R$.

Ex. 5 section 3.5.

The cofactors of the entries of A can be easily and tediously computed, and we get:

$\begin{bmatrix} -8 & -12 & 13 \\ 2 & 3 & -5 \\ 2 & 3 & -2 \end{bmatrix}$ (courtesy of TI83Plus).

Ex. 10 section 3.5. Suppose A is a singular matrix (i.e. non invertible). Then, $|A| = 0$. Now, refer to the proof of Theorem 3.10. which says that the product between A and its adjoint is the identity matrix multiplied by the determinant of A: $A adj(A) = \det(A) I$. Then, if $|A| = 0$ of course $A adj(A) = 0$, the zero matrix.

Ex. 27. We have (with TI83) $\det(A) = 3, \det(A_1) = 3, \det(A_2) = 3, \det(A_3) = 6$, therefore $x_1 = x_2 = 1, x_3 = 2$.

Ex. 37. We have $\begin{vmatrix} k & 1-k \\ 1-k & k \end{vmatrix} = k^2 - (1-k)^2 = 2k-1$.

For $k = \frac{1}{2}$ the two equations represent parallel lines, and therefore the system is inconsistent.

For $k \neq \frac{1}{2}$, we may use Cramer's rule to obtain $x = \frac{4k-3}{2k-1}, y = \frac{4k-1}{2k-1}$.

Ex. 45 (review section) We compute the eigenvalues of the matrix:

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ -2 & 3-\lambda & 0 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (1-\lambda)(3-\lambda)(4-\lambda) = 0 \Rightarrow \lambda = 1, 3, 4.$$

For $\lambda = 1, \begin{bmatrix} 0 & 0 & 0 \\ -2 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is the coefficient matrix of the homogeneous system whose solutions are the eigenvectors corresponding to 1. The solution of that system is $z = 0, y = t, x = t, t \in R$. So the eigenvectors are $(t, t, 0)$.

For $\lambda = 3, \begin{bmatrix} -2 & 0 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is the coefficient matrix of the homogeneous system whose solutions are the eigenvectors corresponding to 3. The solution of that system is $z = 0, y = t, x = 0, t \in R$. So the eigenvectors are $(0, t, 0), t \in R$.

For $\lambda = 4, \begin{bmatrix} -3 & 0 & 0 \\ -2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the coefficient matrix of the homogeneous system whose solutions are the eigenvectors corresponding to 4. The solution of that system is $z = t, y = 0, x = 0, t \in R$. So the eigenvectors are $(0, 0, t), t \in R$.