Math 3260 Spring 2008

Quiz 5(form A) answer key

1. Recall that the set of rational numbers is $Q = \left\{ \frac{m}{n} : m, n \in Z, n \neq 0 \right\}$. The set of rational numbers is not a subspace of R since it is not closed under scalar multiplication. Indeed, if $r = 2 \in Q$ and $c = \sqrt{2} \in R$ scalar, then $c \cdot r = 2\sqrt{2} \notin Q$.

2. We will check that $S = \{(x, 2x) : x \in R\}$ is closed under both vector addition and multiplication by a scalar.
   Let $(x, 2x), (y, 2y) \in S$, $c \in R$. Then
   $(x, 2x) + (y, 2y) = (x + y, 2(x + y)) \in S$ and $c(x, 2x) = (cx, 2cx) \in S$, proving that S is a subspace of R^2.

3. We need to decide if there exist $c_1, c_2 : c_1(1,0,3) + c_2(2,0,-1) = (4,0,6)$. This equation leads to a system of linear equations in c_1, c_2 whose augmented matrix is
   $A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \\ 3 & -1 & 6 \end{bmatrix}_{R_3 - 3R_1} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \\ 0 & -7 & -6 \end{bmatrix}$. The solution to the system is $c_2 = \frac{6}{7}, c_1 = \frac{16}{7}$, and

therefore $(4,0,6) \in span\{(1,0,3),(2,0,-1)\}$.