Math 3260 Spring 2008

Quiz 6 answer key

1. Since $\dim(R^3) = 3$ it is enough to prove that the three vectors are linearly independent in R^3. This is equivalent to checking that the matrix whose rows are the three vectors has rank 3.

   Let $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & \frac{1}{4} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $rank(A) = 3$.

2. We write a vector in W as $(b+c, b, c) = b(1,1,0) + c(1,0,1)$. It is then obvious that the vectors $(1,1,0)$ and $(1,0,1)$ span W. The two vectors are also linearly independent since if $b(1,1,0) + c(1,0,1) = (b+c, b, c) = (0,0,0)$, then $b = c = 0$. Therefore a basis for W is $B = \{(1,1,0),(1,0,1)\}$, and $\dim(W) = 2$.