Examples on the Normal Model

Below you will see some model solution to problems on the Normal model (Chapter 6).

Exercise 8. a) Even if RockReady batteries claim to have a longer life on average, without knowing the standard deviation we may buy batteries whose life *varies* a lot, making it possible for the batteries to die after a few hours.

b) We want to decide which battery is more likely to last 8 hours. We compute the z-score of 8 relative to the two brands given:

DuraTunes: $z = \frac{8-11}{2} = -1.5$; RockReady: $z = \frac{8-12}{1.5} = -2.67$.

Now, interpret the two z-scores as an indicator of how likely are the two brands of batteries to **die** within 8 hours. DuraTunes *does not* have an unusual z-score, whereas RockReady *does have* an unusual z-score, therefore RockReady are more likely to last 8 hours.

c) We repeat the computations for 16 hours.

DuraTunes: $z = \frac{16-11}{2} = 2.5$; RockReady: $z = \frac{16-12}{1.5} = 2.67$.

Now we notice an interesting phenomenon: whereas both z-scores are unusual, DuraTunes has a slightly smaller z-score than RockReady, so DuraTunes would be a better bet.

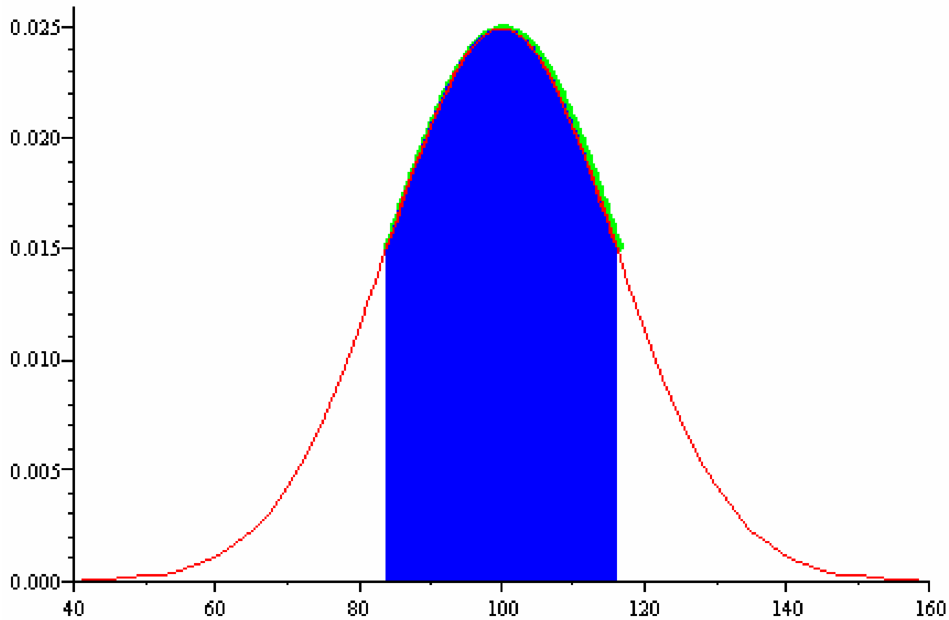
Exercise 10. a) We compute the z-score of the speed limit using the mean of 23.84 and the standard deviation of 3.56. We have: $z = \frac{20-23.84}{3.56} = -1.07$. So, a car going under the speed limit will be *at least* 1.07 standard deviations below the mean.

b) To see which speed is more unusual we compute again the z-scores and decide which z-score is more unusual. A car traveling at 34 mph has a z-score of $z = \frac{34-23.84}{3.56} = 2.85$ and a car traveling at 10 mph has a z-score of $z = \frac{10-23.84}{3.56} = -3.89$. Since -3.89 is farther from zero (the mean of z-scores) than 2.85 it means that a car traveling at 10 mph is more unusual than one traveling at 34 mph.

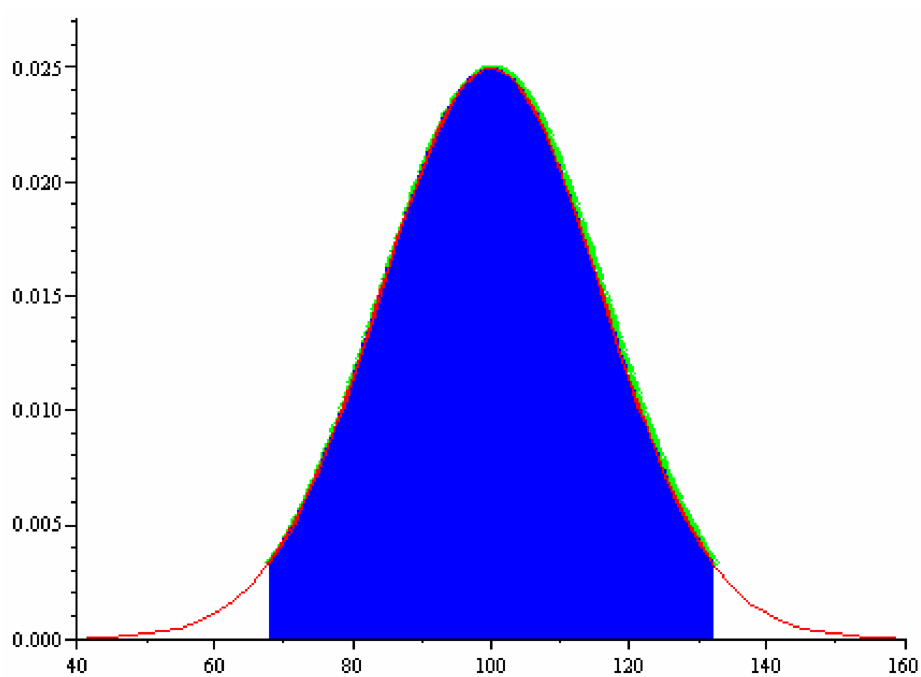
Note: Actually using only our common sense we could have answered that it is more likely to observe cars speeding than slowing down, but I am glad that our intuition is confirmed by the z-scores.

Exercise 16. This problem is asking you to use nothing but the interpretation of the standard deviation and some common sense. So they tell us that on average there were 21,389 fans. Now, a standard deviation of 20 is unrealistic since we expect that the number of fans in the audience would vary a lot more. A standard deviation of 20,000 is again unrealistic: this would mean that they may have played with empty stadiums or very large stadiums as well. Most likely I would believe to be 2,000.

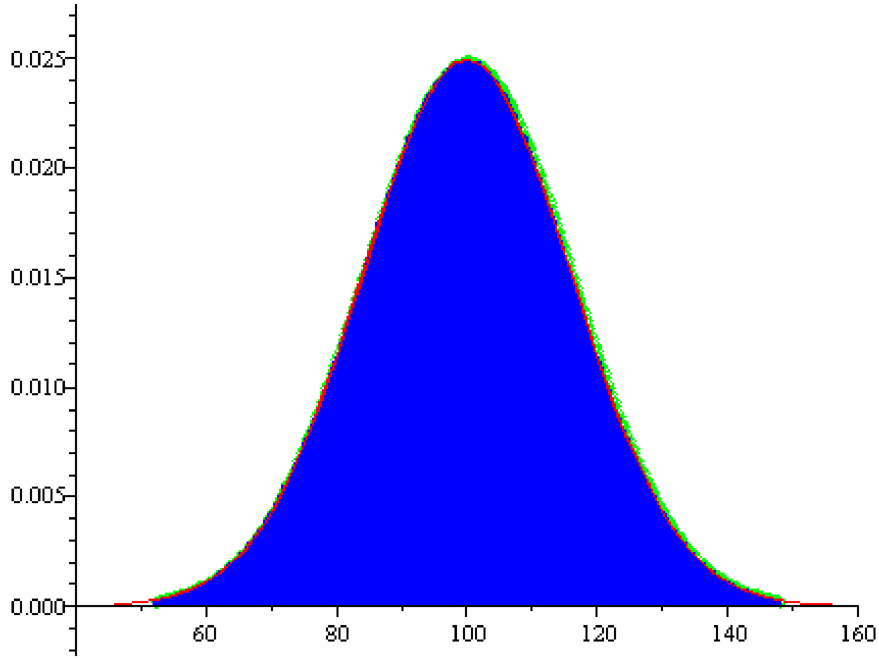
Exercise 18. The figure below shows the region under the Normal model which is one standard deviation within the mean, therefore these are the IQ scores between 100-16=84 and 100+16=116.



The next one shows the region under the Normal model which is two standard deviations within the mean: 100-32=68 to 100+32=132.

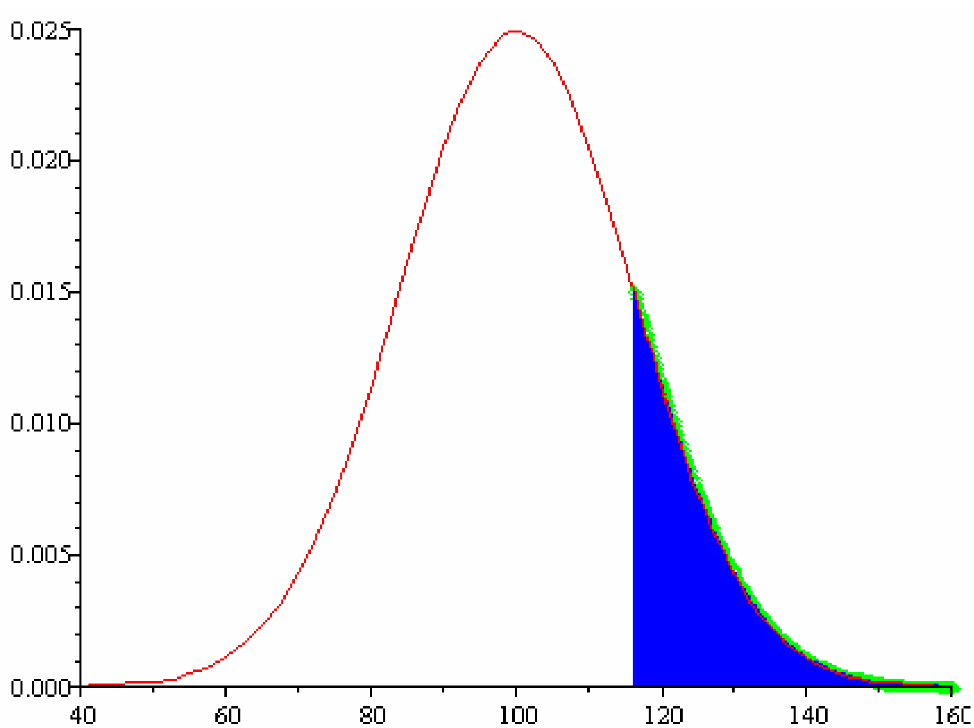


The region below represents the IQ scores which are within three standard deviation from the mean: 100-48=52 to 100+48=148.



b) A we can see from the second figure above, the central 95% of scores are between 68 and 132.

c) The percentage of IQ scores above 116 is given by the area of the shaded region below.



Now, from the Normal model we know that the central 68% of IQ scores are between 84 and 116. Because of the symmetry of the model, the regions to the right and left of the mean, 100, have equal area. Since the total area under the curve is 1, the area to the left of 100 is 0.5. Again, from the symmetry of the mode, the region between 100 and 116 has an area equal to half the area between 84 and 116. Then that area must be half of 68%, or 34%.

So, we have 50% of the area to the left of 100 and another 34% between 100 and 116: this gives us a total area of 84% to the left of 116. Then the region above, covering IQ scores above 116, has an area of 1-0.84=0.16.

Therefore, the percentage of IQ scores above 116 is 16%.

d) We proceed as above: between 84 and 100 there are 34% of IQ scores (half of 68%), between 68 and 100 there are 47.5% of scores (half of 95%). Then, between 68 and 84 there are 47.5%-34%=13.5%.

e) Since between 100 and 132 we have 47.5% of IQ scores, and to the left of 100 we have 50% of IQ scores, it means that there are

$$100\%-(50\%+47.5\%)=2.5\%$$

IQ scores above 132.