MATH 1107

Solutions to Practice test #3

1. Thinking that the 4 individuals were randomly polled so their opinions were independent, then we can compute the probability using independent events. Therefore, P(all four were worried)= $.4^4 = 0.0256$.

2. You can compute the mean and the variance by inserting the data in two lists in the TI83 and use Stat-Calc-1VarStats. Note that the calculator will give you the standard deviation, therefore you will have to take its square to find the variance. I will show the computations according to the formulae:

$\mu = \sum xp(x) = 0\cdot 0.2 + 1\cdot 0.15 + 2\cdot 0.3 + 3\cdot 0.3 + 4\cdot 0.05 = 1.85$

$\sigma^2 = \sum x^2 p(x) = 0^2\cdot 0.2 + 1^2\cdot 0.15 + 2^2\cdot 0.3 + 3^2\cdot 0.3 + 4^2\cdot 0.05 - \mu^2 = 1.4275$

3. We define the random variable *X* to denote the farmer's profit or loss. We have the following model for *X*:

X	−50000	100000
p(x)	0.35	0.65

Then the expected value of X is $\mu = E(X) = -50000\cdot 0.35 + 100000\cdot 0.65 = 47500$.

4. P(at most two classes)= $\frac{36}{90} = 0.4$, and P(at least 3 classes)= $\frac{54}{90} = 0.6$.

5. Problem 15: This problem is like problem 1, so you should know what to do. We get $\mu = 1.7, \sigma = 0.9$.

6. Problem 18: Here is the model for the profit (call it X) on the policy:

X	100-10000	100-3000	100
p(x)	$\frac{1}{2000} = 0.0005$	$\frac{1}{500} = 0.002$	0.9975

Then $\mu = 89, \sigma = 260.54$

7. Problem 29: Assuming that the repair calls for any hour are independent of the other hours, we have that for 8 hours the expected number of repair calls will be $\mu = 8\cdot 1.7 = 13.6$, and the standard deviation is $\sigma = 0.9\cdot\sqrt{8} = 2.55$.

8. Problem 31: a) The standard deviation is so large because the amount varies from a profit for the insurance of \$150 (if there is no fire) to losses which can get quite large in case of damage by fire.

b) If two policies are written, then assuming that they are independent, the mean and standard deviation of the profit are: $\mu = 2\cdot 150 = 300, \sigma = 6000\cdot\sqrt{2} = 8485.28$.

c) If 10000 policies are written, then the mean profit and standard deviation become:

$$\mu = 10000\cdot 150 = 1,500,000, \sigma = 6000\cdot\sqrt{10000} = 600,000$$

If we look at the values for mean and standard deviation, the company is more likely to be profitable since the mean is larger than twice the standard deviation. Thinking that common values of a variable are more likely to be seen within two standard deviations of the mean, it follows that even if we think of having values of the total profit two standard deviations below the mean, those values will still be positive, so the company will be profitable. Notice that this was not the case when only one or two policies were sold.

9. Problem 19: We have a binomial model with $n = 6, p = 0.7$. If we denote by X the number of first serves in, we have:

a) $P(X = 6) = binompdf(6,.7,6) = 0.1176$

b) $P(X = 4) = binompdf(6,.7,4) = 0.3241$

c) $P(X \geq 4) = 1 - P(X \leq 3) = 1 - binomcdf(6,.7,3) = 0.7443$

d) $P(X \leq 4) = binomcdf(6,.7,4) = 0.5798$

10. Problem 25. If we denote by X the number of seats for right-handed students, then X follows a Binomial model with $n = 188, p = 0.87$. Now, there are 170 seats for right-handed students, so a right-handed student will have to use a left tablet if the number of right-handed students is above 170. So we compute:

$$P(X > 170) = 1 - P(X \leq 170) = 1 - binomcdf(188, 0.87, 170) = 0.0611$$

11. Problem 26. We will look at the number of passengers showing up out of the 275 tickets sold. I we imagine that the tickets were sold independently, then the number of passengers showing follows a binomial model with $n = 275, p = 0.95$. Now, the airline will not have enough seats and will have to give away vouchers if the number of passengers showing at the airport is 266 or above. So we compute

$$P(X \geq 266) = 1 - P(X \leq 265) = 1 - binomcdf(275, 0.95, 265) = .1155$$

Based on our result, we can say that 11.55% of the time the airline will have to give away vouchers if they sell 275 seats instead of 265.

12. Problem 13: The sample proportion in this sample is $\hat{p} = \frac{522}{603} = 0.866$. We will decide whether this is an usually high proportion, if we assume that the national retention rate is 74%. If the assumptions are met, the Central Limit Theorem gives us that the sample proportion follows approximately a Normal model with mean $p = 0.74$ and standard deviation $\sigma = \sqrt{\frac{.74 \cdot .26}{603}} = 0.0179$. Since the sample is large enough and both $np = 446.22 \geq 10, n(1-p) = 156.78 \geq 10$, the assumptions are met so we can work with the Normal model. We have: $P(\hat{p} \geq 0.866) = normalcdf(.866, 1, .74, .0179) = 9.7 \cdot 10^{-13}$. With such a small probability it means it is highly unlikely to impossible to observe such a high retention rate, therefore this is a very unusual event, so the college has all reasons to brag about their retention rate.

13. Problem 15: if 400 voters are polled, the newspaper will predict defeat if fewer than half of the voters will approve the budget. Therefore we look at 199 or less votes in favor. In terms of sample proportion this means $\hat{p} \leq \frac{199}{400} = 0.4975$. We want to compute

$P(\hat{p} \le 0.4975)$ using the Central Limit Theorem. We first check that the conditions are met: we have a large sample and with $p = 0.52$ both $np = 208 \ge 10$, $n(1-p) = 192 \ge 10$.

Then, $P(\hat{p} \le 0.4975) = normalcdf(0, 0.4975, .52, \sqrt{\frac{.52 \cdot .48}{400}}) = 0.1839$.

14. Problem 19: The sample is large (n=120). Now with a proportion of $p = 0.6$, the conditions of working with the Normal model are met: $np = 72 \ge 10, n(1-p) = 38 \ge 10$. Since the sample proportion follows approximately a Normal model with mean $p = 0.6$ and standard deviation $\sigma = \sqrt{\frac{.6 \cdot .4}{120}} = 0.0447$, the sample proportion will fall within three standard deviation of the mean 99.7% percent of the time, so if we want to be very sure that all non-smoking customers will be satisfied we can plan for having as many as $p + 3\sigma = 0.6 + 3 \cdot .0447 = 0.7341$ percentage of non-smokers. Therefore we can have about 88 seats for non-smokers.

15. Problem 26: a) Let X denote the amount of rain (in inches). We are told that X follows a Normal model. $P(X > 40) = normalcdf(40, E99, 35.4, 4.2) = .1367$.

b) Now this is an inverse problem: $InvNorm(.20, 35.4, 4.2) = 31.87$. So less than 31.87in of rain fall in the driest 20% of the years.

c) If we look at the average amount of rain for the 4 years, it follows a Normal model with mean 35.4 and standard deviation $\sigma_{\bar{Y}} = \frac{4.2}{\sqrt{4}} = 2.1$.

d) $P(\bar{Y} < 30) = normalcdf(0, 30, 35.4, 2.1) = 0.005$.

16. Problem 35: a) We don't know what the original distribution of carbon monoxide looks like, but we have a large sample, so the Central Limit Theorem applies. The mean Co level follows a model which is approximately Normal with mean 2.9 g/mi and standard deviation $\sigma_{\bar{Y}} = \frac{0.4}{\sqrt{80}} = 0.0447$.

b) $P(3 < \bar{Y} < 3.1) = normalcdf(3, 3.1, 2.9, 0.0447) = 0.0126$.

c) We need to find the 95th percentile of the distribution of the mean CO levels. $InvNorm(.95, 2.9, .0447) = 2.97$