A list of needful things

- Addition rule: $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
- Conditional probability: $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$.
- Events A and B are independent if $P(A\cap B)=P(A)\cdot P(B)$, or equivalently $P(A\mid B)=P(A)$, or $P(B\mid A)=P(B)$

Probability distributions:

	Discrete	Continuous
Probability density function (pdf)	$p(x)\ge 0, \sum_x p(x)=1$	$f(x)\ge 0, \int_R f(x)dx=1$
Cumulative distribution function (cdf)	$F(x)=P(X\le x)=\sum_{y\le x} p(y)$	$F(x)=P(X\le x)=\int_{-\infty}^{x} f(t)dt$
Expected value	$\mu=E(X)=\sum_x xp(x)$	$\mu=E(X)=\int_R xf(x)dx$
Variance	$\sigma^2=E(X^2)-\mu^2=\sum_x x^2p(x)-\mu^2$	$\sigma^2=E(X^2)-\mu^2=\int_R x^2f(x)dx-\mu^2$
Standard deviation	$\sigma=\sqrt{\sigma^2}$	$\sigma=\sqrt{\sigma^2}$
Moment generating function (MGF)	$M(t)=E(e^{tX})=\sum_x e^{tx}p(x)$	$M(t)=E(e^{tX})=\int_R e^{tx}f(x)dx$

Computing with the MGF

$E(X)=M'(0)$

$E(X^2)=M''(0),\ldots,$ in general $E(X^n)=M^{(n)}(0)$

whenever the derivatives of the moment generating function exist.

$\sigma^2=M''(0)-\left[M'(0)\right]^2$.

Binomial(n,p)	$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$	$E(X)=np$	$V(X)=np(1-p)$	$M(t)=$
Hypergeometric	$P(X=k)=\dfrac{\binom{M}{k}\binom{N-M}{n-k}}{\binom{N}{n}}$	$E(X)=n\dfrac{M}{N}$	$V(X)=n\dfrac{M}{N}\dfrac{N-M}{N}\dfrac{N-n}{N-1}$	
Poisson(λ)	$P(X=k)=e^{-\lambda}\dfrac{\lambda^k}{k!}$	$E(X)=\lambda$	$V(X)=\lambda$	$M(t)=$
Uniform(a,b)	$f(x)=\dfrac{1}{b-a}, a\le x\le b$	$E(X)=\dfrac{a+b}{2}$	$V(X)=\dfrac{(b-a)^2}{12}$	

Exponential(λ)	$f(x)=\frac{1}{\lambda}e^{\frac{-x}{\lambda}}, x>0$	$E(X)=\lambda$	$V(X)=\lambda^2$	$M(t)=$
Normal(μ,σ)	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty<x<\infty$	$E(X)=\mu$	$V(X)=\sigma^2$	$M(t)=$

Note: In the above, **"**exp" stands for exponential

Joint Probability Distributions

	Discrete	Continuous
Joint pdf	$p(x,y)\geq 0, \sum_x\sum_y p(x,y)=1$	$f(x,y)\geq 0, \iint_D f(x)dxdy=1$
Marginal of *X*	$P(X=x)=f_X(x)=\sum_y p(x,y)$	$f_X(x)=\int f(x,y)dy$
Marginal of *Y*	$P(Y=y)=f_Y(y)=\sum_x p(x,y)$	$f_Y(y)=\int f(x,y)dx$
Independence of *X* and *Y*	$p(x,y)=f_X(x)f_Y(y)$	$p(x,y)=f_X(x)f_Y(y)$

Linear Combination of Random Variables

Let *X* and *Y* be random variables and *a*, *b* real numbers.

$$E(aX+bY)=aE(X)+bE(Y)$$

If *X* and *Y* are **independent** then $V(aX+bY)=a^2V(X)+b^2V(Y)$.

If *X* and *Y* are independent and *u*, *v* are functions, then $E(u(X)v(Y))=E(u(X))E(v(Y))$.

- If *X* is binomial(*n*,*p*) and *Y* is binomial (*m*,*p*), and X and Y are independent, then *X*+*Y* is Binomial $(n+m,p)$.
- If *X* and *Y* are independent Poisson random variables with means λ_1,λ_2 respectively, then *X*+*Y* is Poisson($\lambda_1+\lambda_2$).
- If *X* and *Y* are independent normally distributed random variables, $N(\mu_1,\sigma_1^2)$, and $N(\mu_2,\sigma_2^2)$ respectively, then $aX+bY$ is $N(a\mu_1+b\mu_2, a^2\sigma_1^2+b^2\sigma_2^2)$.
- More general, if $X_1,X_2,...,X_n$ are independent normal random variables, $X_i \sim N(\mu_i,\sigma_i^2), then \sum_{i=1}^n a_iX_i \sim N(\mu,\sigma^2), \mu=\sum_{i=1}^n a_i\mu_i, \quad \sigma^2=\sum_{i=1}^n a_i^2\sigma_i^2$
- If *X* is $N(\mu,\sigma^2)$ then $Z=\frac{X-\mu}{\sigma}$ is standard normal.
- If *Z* is standard normal, then Z^2 is $\chi^2(1)$ Chi square with one degree of freedom.
- If $Z_1,...,Z_n$ is a random sample from the standard normal distribution, then

$$W=Z_1^2+Z_2^2+...+Z_n^2 \text{ is } \chi^2(n)$$

- If $X_1,...,X_n$ are independent, normally distributed, $N(\mu,\sigma^2)$ and $S^2=\sum_{i=1}^{n}\frac{(X_i-\bar{X})^2}{n-1}$, then $\frac{(n-1)S^2}{\sigma^2}$ is $\chi^2(n-1)$.
- If $X_1,...,X_n$ are independent, normally distributed, $N(\mu,\sigma^2)$ then $T=\frac{\bar{X}-\mu}{S/\sqrt{n}}$ has Student t-distribution with $n-1$ degrees of freedom.
- If $X_1,...,X_n$ are independent, normally distributed, $N(\mu_i,\sigma_i^2)$ then

$$W=\sum_{i=1}^{n}\frac{(X_i-\mu_i)^2}{\sigma_i^2} \text{ is } \chi^2(n).$$

The Central Limit Theorem

Let $X_1,...,X_n$ be a random sample from a distribution with mean μ and standard deviation σ. Then $\bar{X}=\frac{\sum_{i=1}^{n}X_i}{n}$ is approximately $N(\mu,\frac{\sigma^2}{n})$.

In particular the result applies for a sum of a random sample: $T=\sum_{i=1}^{n}X_i$ is approximately normally distributed with mean $E(T)=n\mu$ and variance $Var(T)=n\sigma^2$.