My Philosophy of Mathematics Education

Jonathan Lewin
## Contents

**Prologue**  
- The Role of this Document  
- What Is This Document About?  

1 **Overcoming the Fear**  
1.1 The Role of Problem Solving  
1.2 Avoiding Sudden Death Situations  

2 **Helping Students Study**  
2.1 The Need for Integrity in the Description of Course Content  
2.2 The Role and Nature of Lecture Notes  
2.3 Student-Teacher Interaction  
2.4 The Need for Precise Language  
2.5 The Link between Teaching and Learning  

3 **The Use of Technology in the Teaching of Mathematics**  
3.1 Overview  
3.2 The Two Roles of Technology  
3.3 The Role of Computer Algebra Systems  
3.4 Technology As a Means of Communication  

4 **Application of Mathematics to Other Disciplines**  
4.1 Introduction  
4.2 Some Illustrative Examples  
4.3 Exponential Growth and Decay
Prologue

The Role of this Document
I have been asked to supply a description of my teaching philosophy and am happy to comply with this request. Please understand that I am cramming the absolute minimum that I feel I can write without trivializing or distorting my thoughts on the subject. The contents of this document represent my constant accumulated thoughts gathered over a period of some 40 years and I regret that I am unable to squeeze what I have to say into three pages.

The spirit in which I am supplying this document is that I hope that the communication between instructors that our statements may generate will serve to improve the quality of our instruction at Kennesaw State University and, perhaps, in the mathematical community in general.

What Is This Document About?
Mathematics is a very difficult discipline. In addition to being hard and demanding, it is also intimidating. Everyone, from those who see themselves as refugees from mathematics to those who occupy the most distinguished positions in the mathematical community, all regard mathematics with a healthy respect tinged with not a little fear.

Thus, those of us who have the responsibility of imparting mathematics to others, share an awesome responsibility. We are faced with a very large number of students who enter each course wondering if that particular course will turn out to be the killer that will push them out of their chosen direction of academic travel. They are aware that a difficult task lies before them and, moreover, they are denied the comfort of knowing that, if they do their best to perform that task, then all will be well. They know that there are no guarantees. Yes, there is mathematics anxiety; and it is not a disease. There is nothing wrong with a feeling of anxiety when the fear that generates it is well founded.

A major feature of my teaching philosophy, as I shall describe it below, deals with this fear. I would like to make the point from the onset that there are no lazy students. Failure on the part of a student to engage in his/her study,
to do assigned reading or to do assigned homework can be due to lack of time or energy arising from the student’s busy lifestyle; but the greatest cause, by far, is the student’s feeling of fear, fear of not knowing where to begin and fear that the effort and pain involved in the study process will not necessarily bear fruit. In many circumstances, students discover that they feel less depressed when their mathematics books are closed.

Over the past 40 years, during which time I have been teaching mathematics at various levels (from high school to graduate school), I have agonized over this problem. There is no simple solution; and I certainly do not believe that we should lie to our students by telling them that mathematics is easy and that learning is a thrilling experience. The students are smart enough to know when their instructors are lying; and we should not begin our relationships with them by losing credibility. Instead, we should be open about the difficult and sometimes frightening task that lies ahead. We should make it clear that success in this life comes to those who are willing to persevere with a task even when they are not having fun, that instant gratification is not always possible, but that the long term benefits and rewards are well worth the investment of time and energy that we are asking them to make.

If I have to place the heart of my philosophy in a nutshell then I have to say that the fear is there and it is real. The job of a competent mathematics instructor is to reduce that fear, to present and examine material in such a way as to minimize the dangers and give the students as much confidence as possible that the study process will bear fruit. All this must be done without compromising the goals that have to be met in mathematical study; especially the goal of mathematical understanding. As a matter of fact, awareness on the part of a student that he/she has achieved understanding of a mathematical idea may go a long way towards helping to evaporate the fear of mathematics. Indeed, the one litmus test of successful mathematics instruction is the student’s feeling of pride, enjoyment of the sense of mastery of a mathematical idea and an aesthetic appreciation of its beauty.
Chapter 1
Overcoming the Fear

1.1 The Role of Problem Solving
The essence of the kind of thinking required of a professional mathematician is that it is creative; that it produces ideas, new mathematical theories and solutions to hitherto unsolved problems. The kind of thinking required of those who use mathematics in the sciences, technology and economics may be similar; also requiring the solution of problems to which the individual has not previously been exposed. One might think, therefore, that this sort of theme should be the basis of many, if not most, mathematics courses. Indeed, there are significant movements that favour this aspect of mathematics education. Those “Moore method” courses that are so popular in some schools come to mind. So do those “problem solving” courses, the Putnam examination, the classical English Tripos of yesteryear and all those olympiads and tournaments that are so popular in some quarters.

Indeed; my own undergraduate education was peppered with this sort of thing at the University of the Witwatersrand several decades ago. Thirty percent of every examination in mathematics was devoted to questions that required a certain level of on the spot original thinking on the part of the candidate. On several occasions I found my nose starting to bleed when they were preparing to distribute those question papers.

I promised myself that I would never inflict this sort of thing upon my own students. Yes; I agree that the ability to come up with solutions to problems is an important constituent of mathematical knowledge. However, I do not believe that an examination is capable of measuring a student’s future potential as a problem solver and I do not believe that the solving of unseen problems should be the central focus of an undergraduate mathematics course.

1.1.1 Difficulty of Measurement of Problem Solving Skills
Solutions to problems come when they are least expected. The great mathematician Littlewood used to say that, when we was working on a problem, he would go walking in the countryside. In my own case, the best piece of mathematics I have ever done (the solution to a problem that had bothered
me for some ten years) was achieved out of the blue when I was stopped at a red traffic light. I solved another important problem in the shower one day after swimming (which proves that a clean mathematician may be a good mathematician), another while sitting alone, shivering in the dark outside my cousin Zoe’s house because one of her other guests was smoking indoors. I have never solved a major problem when I actually sat down to do so.

From time to time, I have composed nice musical melodies; always unconsciously. I find myself humming a new melody and write it down. But I could never sit down to compose one. Perhaps I don’t have the talent. So it is with mathematics. I have no reason to believe that the student who will come up with something clever in an examination is necessarily the student who will come up with some earth shattering discovery one day.

So I see no reason to inflict that pain upon my students. I promise them that I am looking only for understanding of the problems they have been given to study; that my examinations will contain no tricks, no surprises and no calls for “think on your feet” creativity.

In this way, I am able to give my students more confident that, if they work hard studying the material and doing the work of the course, then their work will be rewarded.

1.1.2 The Profile of Problem Solving During the Study Process

I do not ignore the undoubted importance of problem solving as a skill to be acquired during one’s mathematical education. However, I also believe that there are severe limits to the effectiveness of the kind of training that is geared specifically for problem solving. In other words, I think that an abstract set of instructions that suggest a general strategy for the solution of problems has very limited value.

Instead, I believe that the greatest tool for the solution of mathematical problems is an intimate understanding of the solution of other problems. I believe that a very substantial portion of undergraduate mathematical study should be the study of well written solutions to problems until such solutions are understood well.

Again, there is a musical analogy: If you want to study musical composition, they start you off studying the work of Johann Sebastian Bach.

Thus, a significant part of my own mathematics courses dwells upon presentation of solved problems which I expect my students to learn how to write
1.2 Avoiding Sudden Death Situations

out, closed book, in such a way as to demonstrate that they are achieving understanding of the techniques that the problems require.

1.1.3 My Approach to Homework

While many instructors may assign homework and, at some later stage, provide solutions to some or all of the problems, I do it the other way around. I provide the solutions and then ask the students to study them by writing them out. I then follow these problems by others that are very similar and ask the students to write these out on their own. However, even for the latter problems, I make it clear that I am perfectly willing to provide the solutions at their request.

My viewpoint of homework is totally incompatible with the notion that a student’s performance in homework should be an ingredient in the assessment of his/her grade. I make it clear that the doing of homework is part of the study process; that the student is doing the homework for him or her self; not for me. I make it clear that the homework that I want to see is the homework that is wrong, so that I can correct student errors, or incomplete, so that I can show the student how to complete it. I see no point in looking at material that they know to be correct. My feeling is that if a man has gout in one foot, he achieves nothing by going to the doctor and showing his other foot.

I do, however, keep my eyes open for those (superior) students who feel a need for challenging unseen problems and I see to it that such problems are always available. But I am careful not to impose those problems on the rank and file. Nor do I make such problems a part of the way I assess student performance.

My approach has done much to reduce the fear of mathematics because the core of this fear is rooted in the concern that one may be called upon to produce the solution to a problem that one has never seen before and that the solution may prove to be elusive while the clock ticks the time mercilessly away. That will not happen in my examinations to anyone who has done the work of the class.

1.2 Avoiding Sudden Death Situations

In my view, a mathematics examination should not designed to determine the extent to which a candidate has become qualified to be an instructor for the course. There is no doubt that if I, the professor, were to find even one problem that I am unable to solve in a course that I am teaching, then my
grade as an instructor must be a resounding F. However, it would be wrong and unfair to hold students to such an exacting standard.

1.2.1 My Disapproval of Sudden Death Examinations

In my view, the primary purpose of a mathematics examination is to determine whether the student has acquired that critical mass of knowledge and understanding which, in the instructor’s opinion, will provide a sufficient basis for further study or application of the material to other fields. In my view, it is a simple fact of life that students; particularly those students who are supporting themselves and particularly those students who are taking many courses of study simultaneously, will not normally find the time and energy to study all the material that was presented in the classroom. They have made a first pass through the material. We have to ask whether they have done enough to enable them to work with what they have done and to make a second pass through the material on their own if and when the need for that second pass arises.

Thus, I disapprove of examinations that are designed to probe the student for gaps in his/her knowledge. In my view, the purpose of a mathematics examination should be to provide the candidate with the opportunity to demonstrate that he/she has acquired skills, knowledge and understanding of some of the course material and that the depth and breadth of this understanding does indeed constitute the required critical mass to which I have referred.

I believe, therefore, that quality tells us much more about a student than quantity. I provide a very generous choice of questions in my examinations; a choice that lets each student avoid those areas that he/she finds most troubling. The other side of the coin, however, is that, once a student has selected the material to be answered, I expect quality. I expect a student to be writing something down because it is what he/she wants to write; not because it is what he/she believes that I, the examiner, want to read. I expect my students to write in meaningful complete sentences using correct mathematical notation and I expect them to write with conviction and, if possible, with enthusiasm and even passion.

1.2.2 My Disapproval of Sudden Death Assignment of Letter Grades

I disapprove of the traditional 90/80/70/60 cut-off scheme for the assigning of letter grades. I am outraged at the thought that a student who achieves 55% in an examination should be categorized as equal to one who is unable
to do anything at all. As a matter of fact, a student who achieves 55% in an examination has demonstrated mastery of more than half of what he/she was required to write. While such a score should not bring glory and honour, it does, nevertheless, represent measurable achievement and that score is my minimal C.

I give A for 85%. I give B for 70%. I give C for 55%. I give D for 40%.

In spite of my routine, no-tricks examinations, my provision of a wide choice of questions and my gentle grading curve, I am not what one might call a pushover. Although the students know that it is very easy to receive an A in my courses, they also know that it is equally easy to receive an F. Yes, there is a choice of questions, but my message is: You are still required to know something and to produce quality in what you write. Once you have made your choice, you must demonstrate that you are writing on your favourite topics.
Chapter 2
Helping Students Study

2.1 The Need for Integrity in the Description of Course Content

I have said that I believe that a principal reason for lack of adequate student performance is the student’s fear of the study process. One of the ways in which this fear is aggravated is the time honoured habit of mathematics instructors of presenting material according to a double standard. Some material is presented as a fully fledged component of the course while other material is presented, as if for the record, in a half hearted fashion and in such a way that it is not necessarily meant to be understood and will certainly not appear in the examinations. Students have the task of distinguishing between the two types of course material, so that they can discard the material that doesn’t belong. They will study whatever they are required to study; no more.

I believe that a good mathematics instructor will present a much more honest profile of a course. A good instructor ought to make a clear decision of what can be covered meaningfully at the level of a given course, with the students who have entered it and in the time that is available for the study of the material. If an item cannot be presented in the classroom in such a way that the students can reasonably be required to master it and can be expected to demonstrate that mastery in the examination, then the item should not be covered at all. It is better to cover a smaller syllabus honestly and meaningfully than to give the appearance of covering more material while covering some of that material fraudulently.

The litmus test of whether or not an item has been covered is whether or not it can appear in the examination. If it can’t, then it wasn’t covered. Students should be told exactly what they are expected to do, that expectation should be reasonable and the task should be attainable. Students should know that all material seen in the classroom is to be studied and they should be able to trust their instructor to have presented the material in such a way that it really can be studied properly and understood.
2.2 The Role and Nature of Lecture Notes

Although the text adopted for a given course may, at times, serve as a useful source of reference material, it cannot often serve to define the actual course contents. The lecture notes must do that. Therefore, regardless of the role of the text book, students must possess a complete, reliable and readable set of lecture notes that document the actual proceedings of the classroom and that define precisely what was covered and, therefore, what the students will be expected to know in the examination. A list of subject headings is woefully inadequate for this purpose. Only a complete set of lecture notes can display the fine tuning; show exactly what kind of problem is being solved and by what approach and where the emphasis and de-emphasis lies. Only a student who possesses a good set of lecture notes can engage in serious meaningful study of the course material.

In my view, it is an instructor’s duty to ensure that every student acquires a complete, reliable and readable set of lecture notes. Furthermore, I do not believe that students can be expected to write those notes themselves in the classroom while also trying to take in the ideas being presented. My experience is that notes that students write for themselves during the lecture are woefully inadequate and contain many serious errors and omissions; even when the instructor wrote everything carefully on the blackboard. Thus I believe that a good instructor should provide the lecture notes to the students in another way.

In my own case, I write my lecture notes while I am teaching in the classroom and then, at the end of the lecture, I drag the material straight into my website. This process takes only a couple of minutes and, once it has been completed, the students can open the notes and print them. In my case, the notes are created in my laptop computer and projected onto a screen during the lecture. Some instructors use other devices, such as the SmartBoard. At any given time, my students can find all of the lecture notes that have been given to date in their course. Furthermore, those in need of review of material belonging to earlier courses can often find that material in my other course folders.

Some instructors may choose to create their notes and then print them for the students but, in my view, there are some serious dangers to be avoided:

1. If the notes are created on the blackboard at the time of the lecture then the instructor has the job of making a faithful copy to provide to the students.
2. If the notes are to be created before the lecture then the instructor has the job of knowing ahead of time exactly what will take place in the classroom. Even after 40 years of teaching mathematics, I still can’t do that. Whatever I write beforehand, I always find that when I’m actually talking to the students, I want to say something else. There is something about that eye to eye contact that nothing else can replace.¹

3. I strongly disapprove of the practice adopted by some instructors of making up notes in advance on transparencies and then displaying them to students with an overhead projector. An overhead projector should be used very sparingly. Examples of valid use of an overhead projector are the display of a list of scientific or statistical data or the display of a figure that would be hard to produce in the classroom. In my view, any other use of an overhead projector is very questionable.

My disapproval of the process of displaying ready written notes in a transparency arises from my strong belief that a good lecture must be spontaneous. The material must be in the process of being born as it is explained and the instructor must be producing it. Nor, in my opinion, should a mathematics instructor be making use of his/her own personal notes in order to give the lecture. The lecture should come out of the instructor’s mind, except (as I have said) for lists of data and for complicated figures.

An instructor who has to use lecture notes while standing before a class of students gives the message: I’m reading from my notes because I don’t really know this stuff and, if I don’t know it, then you can bet that you never will either. I have never seen a soloist at a concert with music manuscript before him/her; and the principle is the same.

4. I am presently experimenting with the process of making sound movie versions of my lecture notes available alongside the standard printed form. The movie version will show the same computer screen (“blackboard”) that appeared in the actual lecture and will play a recording of everything I said during the class. I have several technical problems to overcome before this feature can be made available.

### 2.3 Student-Teacher Interaction

Students learn very rapidly that the only thing harder than answering a math-

¹ That is why, I believe, the process known as distance learning suffers from very severe limitations.
2.4 The Need for Precise Language

In my opinion, it is much harder to talk nonsense than it is to say something meaningful. Mathematics becomes harder, not easier, when the language used by student and instructor degenerates into meaningless sounds and symbols.

Sadly, our community has not adhered sufficiently to this principle and our literature is full of syntax and presentation errors which, had they appeared in a basic course on English writing, would have led to some considerable action by the red pen.

When a particular meaningless (or incorrect) phrase has been used often enough, it often acquires a new meaning to the person who is using it. That person tends to forget that each group of students who are learning the material for the first time must learn to reinterpret that phrase. By including invalid language we are increasing the work load of our students. And, even worse, we are teaching them that we are unreliable sources of information.

A fundamental tenet of my teaching philosophy is the principle that the language used by an instructor must be completely correct at all times, that every sentence, whether uttered by mouth, written on a blackboard or written
in other course materials, must be complete and say exactly what the instructor wanted it to say. Everything that we say must be able to serve as an example to our students; to show them what is possible and to give them a goal in their own attempts at expression of their ideas.

Above all, our students need to know that they can trust us. They need to know that everything before them ought to make sense and that, if they can’t understand something, then they need help. As things are at present, students may have to consider the possibility that what they are attempting to read may not have any meaning to understand.

2.5 The Link between Teaching and Learning

We need to make it clear to our students that there is no such thing as understanding of a mathematical problem without the corresponding ability to find the words with which to write out the solution, and to write it well.

We need to make it clear to our students that the moment at which a given mathematical topic or problem is understood is the moment at which one feels eager to write down the explanation and to explain it to others. When a person understands a mathematical problem, the urge to explain it to others is similar to the urge we have to tell a good joke that we have heard, particularly if we are proud of the fact that we understood it.

We need to explain to those of our students who will one day be teachers that, until they have experienced that urge to communicate a given piece of mathematics, they are unqualified to teach it.

We need to make our students understand that one of the greatest tools they have for deepening their understanding of the material we are teaching them is to simulate the process of teaching it to others. I have already said that I believe that students should learn to write out the solution of every item given in the classroom and to do that writing closed book. During that writing, the student should imagine that he/she is the teacher, explaining to an imaginary (or real) audience. Only when one feels the satisfaction of having given a really good and inspired explanation of the material, can one claim to understand the material oneself.

In other words, the acts of learning mathematics and teaching mathematics are precisely the same thing. There is no such thing as a good mathematician who is not an outstanding and inspired teacher; and there is no such thing as a good teacher who does not feel a passion for the mathematics that he/she is teaching and who is not engaged enthusiastically in constant further study.
2.5 The Link between Teaching and Learning

of mathematics. Consequently, I believe that a stated commitment to mathematics education rings hollow unless it is accompanied by an equal or greater commitment to mathematics itself.
Chapter 3
The Use of Technology in the Teaching of Mathematics

3.1 Overview
I write this chapter with mixed feelings. On the one hand, I have been very active in this field. I have written articles. I have made so many presentations at professional conferences that I have lost track of them. I have given seminars and workshops of varying kinds at many academic institutions, high schools, colleges and research universities both in the United States and in other parts of the world. I am one of the editors of an on-line periodical based at Texas A&M University devoted to this topic. In short, in at least some of the aspects of the application of technology in the teaching of mathematics, I am an acknowledged authority.

However, I am also very concerned that technology is being misused in many contexts. I am more than a little concerned at the high profile that the use of technology apparently enjoys with administrators in many academic institutions throughout the world; and I am disturbed to note that teaching faculty have the perception that if they are seen to use technology then the assessment of their job performances will go up by a couple of notches; regardless of the actual value, if any of that technology.

The fact is that, just because the use of technological tools is of profound benefit to us under some circumstances, that doesn’t mean that all use of such tools will benefit us. These tools are like open heart surgery. Open heart surgery saves lives. We are better off, by far, because of its availability. But that doesn’t mean that I need it today. May none of us need it. In other words, I believe that technology should be used only with the utmost caution; particularly at primary and secondary level.

3.2 The Two Roles of Technology
Technology plays two major roles in the teaching of mathematics:
1. Technology provides us with computer algebra systems (and hand held calculators) that allow us to explore mathematics interactively.
2. Technology provides a means of communication between people.

### 3.3 The Role of Computer Algebra Systems

I have a love/hate relationship with computer algebra systems. On the one hand, I have no doubt that the availability of such systems has opened doors to many exciting possibilities that never existed before. I myself have profited by the availability of computer algebra systems in the interactive ingredients of my new book: *An Interactive Introduction to Mathematical Analysis* which is in the process of being published by Cambridge University Press.

However, I would like to include a small excerpt from the preface of that book:

> In spite of the obvious value of interactive reading that makes use of the computing features of Maple, this text does not go out of its way to present interactive reading on every page. Certainly, there are topics in this text for which the use of Maple computing features is relevant and useful, but there are even more topics in which an attempt to use such computing features would be artificial and counterproductive.

> The philosophy of this book is that, where the nature of the material being studied makes the computing features useful, these features should be exploited. However, where the material would not benefit from these computing features, the features have no place. Under no circumstances is the material of this book specifically chosen in order to provide opportunities to use the computing features. In this sense, my book is not a “reform” text.

The undoubted great value of computer algebra systems has, unfortunately, given rise to a fanaticism that, I believe, is very harmful to the cause of mathematics education. In some quarters, the availability of computer algebra systems has produced a generation of button pushers who have scant knowledge of mathematical principles and subminimal skills. And I have seen too many mathematics courses that have been woven around the technological tools; that have been enslaved by them to the extent that the original goals of conveying mathematical understanding have been lost.
3.4 Technology As a Means of Communication

Nothing I can say here could overstate the enthusiasm with which I view the communications role of technology. The blackboard will eventually be obsolete and, instead, instructors will be able to provide high quality course materials to students as these materials are being composed. In another part of this document, I have given a brief description of my own work on this front; work that I have presented to many people in various parts of the world.

I am, with difficulty, resisting the temptation to give the details of this work here.
Chapter 4
Application of Mathematics to Other Disciplines

4.1 Introduction
Mathematics appears to play a dual role in our society. It is a discipline in its own right; but it is also an indispensable tool for work in other disciplines in science, technology and economics. It is to be expected, then, that a university undergraduate mathematics curriculum should be designed in careful coordination with other curricula on the campus, in order to ensure that students studying those other curricula will have the mathematical tools that they require. Such coordination has been the foundation stone of mathematical application to other disciplines for the past hundred years.

In recent years, however, it has also become fashionable to maintain that mathematics courses themselves should include an ingredient of applications work in other disciplines. I regard this notion as a wonderful idea but, like all wonderful ideas, it needs to be approached with caution. The mere fact that an ingredient of applications work may be desirable in some contexts and in some courses does not automatically imply that such an ingredient should be desirable in every course or at every level. Nor does it imply that if some is good then more is better.

In fact, there are some very real dangers involved in the inclusion of applications material in a mathematics course.

1. Time and stamina are in very short supply and any effort that is expended on one topic must necessarily come at the expense of another topic. Therefore, in any decision to include extra material we have to do a very delicate cost-benefit analysis.

2. Depending upon the nature of and level of a particular mathematics course, there may be a danger that applications material may muddy the water and prevent students from developing an appreciation of the principal objective of the course, which is to convey to them an understanding of mathematical principles.
3. Applications material should not be included until the mathematical tools that it requires have been studied properly. Those tools should not have been introduced half heartedly or prematurely with the specific intent of allowing the course to include the applications material. Instead, the tools should have been developed naturally in the course of a program of study to which they truly belong and in which their presence is properly motivated and well understood.

4. An instructor who intends to introduce applications material from disciplines other than mathematics needs to have good reason to believe that the students have sufficient background in that other discipline to enable them to follow the application and, just as importantly, to enable them to appreciate its role in that other discipline. Ideally, students should have made a study of the other discipline and should have come to realize, while making that study, that the proposed application had to be omitted because the required mathematical tools were not available at that time.

4.2 Some Illustrative Examples

I end this description of my teaching philosophy by referring to two examples of applications and the way in which I look upon them.

4.2.1 Applications in a Course on Mathematical Statistics

My understanding of statistics courses is that their subject matter is particularly well suited to applications to other areas and that the value of the methods used can often best be understood in the light of examples of such applications.

Therefore, particularly in view of the wide variety of both hard and soft sciences that make use of statistical methods, I believe that a strong applications focus in statistics courses, both elementary and advanced, may be very desirable or even necessary.

4.2.2 The Hanging Chain Problem

The hanging chain problem can play an interesting role in a course in differential equations. Provided that the students have come into the course on differential equations with sufficient background in the disciplines to which applications can be made, applications made inside a course in differential equations can be very desirable.
I make this statement with some anxiety, however. Suppose, for example, that we wish to consider a heavy flexible chain that is suspended from two points $A$ and $B$ at the same height:

Students who have an acquaintance with elementary classical mechanics and who are familiar with the process of evaluating integrals in order to find the centroid of a geometric object may appreciate how the physical principle; that the chain must hang in such a way that its centre of gravity is as low as possible, leads to a differential equation the solution of which gives the curve in the form

$$y = ae^{bx} + ae^{-bx}$$

(with a suitable choice of axes).

Such an item in a mathematics course could well prove to be useful and informative.

I mention this example specifically because I reviewed an elementary text once (a text originating at Georgia Southern University, I'm sorry to say) that contained an “applications” problem that started by saying that scientists have determined that a heavy flexible chain must hang in a curve of this kind. Then the problem went on to ask questions about a cable in the Golden Gate Bridge; questions that required nothing more than the solution of a set of linear equations. I regard that type of applications problem as fake and totally worthless.

### 4.3 Exponential Growth and Decay

Students who have covered enough calculus to have encountered the natural exponential and logarithmic functions can determine easily that a function $f$
that grows at a rate proportional to its own value must satisfy a condition of the type

\[ f(t) = ae^{bt} \]

for every number \( t \), where \( a \) and \( b \) are suitable constants. The laws of exponential growth and decay follow almost at once and this topic makes for a nice short applications example.

However, I have noticed an alarming tendency in recent years for elementary texts at the precalculus level have begun introducing the natural exponential and logarithmic functions as part of the process of teaching the algebra of logarithms and exponents. The use of natural logarithms in this kind of elementary context is unmotivated and seems to be made only to justify an application to exponential growth and decay problems; an application that is devoid of any explanation. I view this kind of fake application with dismay.