Yours IS to reason: Don’t just invert and multiply!!

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2008 Georgia Mathematics Conference
The slides & handouts are available at:
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GPS

M5N4. Students will continue to develop their understanding of the meaning of common fractions and compute with them.
   d. Model the multiplication and division of common fractions.

M6N1: Students will understand the meaning of the four arithmetic operations as related to positive rational numbers and will use these concepts to solve problems.
   e. Multiply and divide fractions and mixed numbers.
GPS

• As a result, implementation of Georgia’s Performance Standards places a greater emphasis on problem solving, reasoning, representation, connections, and communication.

• Instruction and assessment should include the use of manipulatives and appropriate technology. Topics should be represented in multiple ways including concrete/pictorial, verbal/written, numeric/data-based, graphical, and symbolic. Concepts should be introduced and used in the context of real world phenomena.
Is this all possible?

• What’s wrong with simply memorizing “invert and multiply”?
• What real world phenomena involve division of fractions?
• What multiple representations - that will be helpful for students to make sense of multiplication and division of fractions?
Some Possibilities

• Using measurement division

• Using fair-sharing division: Japanese approach

• Using properties of division
Using a meaning of division

• Two meanings of division:
  – How many (equal) groups can you make?
    \textit{Measurement (repeated subtraction) division}
  – How many are in each group?
    \textit{Fair sharing division}

M3N4 (b)
Two types of division

• Measurement
  There are 24 candies. If you give 6 candies to each person, how many people can get the candies?

• Fair sharing
  There are 24 candies. If 6 people share the candies fairly, how many candies will each person receive?
Is this a division problem?

There are \( \frac{15}{4} \) pounds of flour. A recipe for a loaf of bread calls for \( \frac{3}{4} \) pounds of flour. How many loaves of bread can we make if we use all flour?
How about this one?

There are $2 \frac{2}{5}$ gallons of ice cream. If each banana split needs $\frac{2}{5}$ gallons of ice cream, how many banana splits can you make?
Problem 1

- Rectangles
- Rectangles with diagonal lines
- Rectangles with horizontal lines
Problem 1
Problem 1
Can you solve each of the following using a picture?

\[
\frac{25}{6} \div \frac{5}{6} = ?
\]

\[
\frac{18}{7} \div \frac{3}{7} = ?
\]
What about these?

\[
\frac{18}{89} \div \frac{3}{89} = ?
\]
What about this one?

\[
\frac{9}{4} \div \frac{3}{8} = ?
\]
So, how do we divide a fraction by another fraction?

• If the denominators are the same, simply divide the numerator of the dividend by the numerator of the divisor.

• If the denominators are not the same, change the fractions so that they have a common denominator.

• Go to the first bullet.
What was missing?

- What about something like this one?

\[
\frac{17}{4} \div \frac{3}{4} = ?
\]

- What about “invert-and-multiply”?
Can we develop the invert-and-multiply meaningfully, through contextualized problems?

What makes division (and multiplication) by a fraction difficult?
Multiplication and Division

- Equal Groups

- 3 apples in each pile; 4 piles
- 12 apples altogether
Multiplication and Division

• Equal Groups

• 0.8 oz of juice in each glass
• 4 glasses
• 3.2 oz of juice altogether
Multiplication and Division

- Equal Groups
- $\frac{2}{3}$ groups of $\frac{3}{5}$?
Excursion through a Japanese elementary math textbook

1. With 1 dl of paint you can paint \(\frac{3}{5}\) m\(^2\) of boards. How many m\(^2\) can you paint with 2 dl of paint?

\[
\frac{3}{5} \times 2 = ?
\]
When you multiply a fraction by a whole number, keep the denominator the same and multiply the numerator by the whole number. $rac{b}{a} \times c = \frac{b \times c}{a}$
\[ \frac{a}{b} \times c = \frac{a \times c}{b} \]
With 3\text{dl} of paint you can paint $\frac{4}{5} \text{m}^2$ of boards. How many $\text{m}^2$ can you paint with 1\text{dl} of paint?
When you divide a fraction by a whole number, keep the numerator the same and multiply the denominator by the whole number.

\[ \frac{4}{5} \div 3 = \frac{4}{5 \times 3} = \frac{4}{15} \]

Answer: \( \frac{4}{15} \text{ m}^2 \)
\[ \frac{4}{5} \div 3 = ? \]
\[
\frac{4}{5} \div 3 = ?
\]
$\frac{4}{5} \div 3 = ?$
\[ \frac{4}{5} \div 3 = ? \]

\[ \frac{4}{5 \times 3} \]

\[ \frac{4}{5} \]

\[ \frac{a}{b} \div c = \frac{a}{b \times c} \]
With 1 dl of paint you can paint $\frac{4}{5}$ m$^2$ of boards.
How many m$^2$ can you paint with $\frac{2}{3}$ dl of paint?

What kind of math sentence should you write?

If you use 2 dl of paint, the equation should be...

$\text{Area that can be painted with 1 dl} \times \text{Amount you use (in dl)} = \text{Area you can paint}$
2. How many $m^2$ can you paint with $\frac{1}{3} \text{ dl}$?

Divide $\frac{4}{5} m^2$ into 3 equal parts, and then...

3. How many $m^2$ can you paint with $\frac{2}{3} \text{ dl}$?

Multiply the area you can paint with $\frac{1}{3} \text{ dl}$ by 2, and then...

$$\frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2$$

$$= \frac{4}{5 \times 3} \times 2$$

Answer: $\square m^2$
\[ \frac{4}{5} \times \frac{2}{3} = ? \]
\[
\frac{4}{5} \times \frac{2}{3} = ?
\]
\[
\frac{4}{5} \times \frac{2}{3} = ?
\]
\[
\frac{4}{5} \times \frac{2}{3} = \left( \frac{4}{5} \div 3 \right) \times 2
\]
\[
\frac{4}{5} \times \frac{2}{3} = \frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2 = \left(\frac{4}{5 \times 3}\right) \times 2
\]
\[
\frac{4}{5} \times \frac{2}{3} = ?
\]

\[
\frac{4}{5} \times \frac{2}{3} = \left(\frac{4}{5} \div 3\right) \times 2 = \left(\frac{4}{5 \times 3}\right) \times 2 = \frac{4 \times 2}{5 \times 3}
\]
When you multiply a fraction by another fraction, multiply the two denominators together and the two numerators together. \[
\frac{b}{a} \times \frac{d}{c} = \frac{b \times d}{a \times c}
\]
With \( \frac{3}{4} \) dl of paint you can paint \( \frac{2}{5} \) m\(^2\) of board.

How many m\(^2\) can you paint with 1 dl of paint?

What math sentence should we write?

If 2 dl was used to paint \( \frac{2}{5} \) m\(^2\), the math sentence would be...

Area painted \( \div \) Amount used (dl) = Area that can be painted with 1 dl
2. How many $m^2$ can you paint with $\frac{1}{4} \, dl$?

Divide $\frac{2}{5} \, m^2$ into 3 equal parts and then...

3. How many $m^2$ can you paint with $1 \, dl$?

Multiply the area that you can paint with $\frac{1}{4} \, dl$ of paint by 4, then...

$$\frac{2}{5} \div \frac{3}{4} = \left( \frac{2}{5} \div 3 \right) \times 4$$

$$= \frac{2}{5 \times 3} \times 4$$

Answer: $m^2$
\[
\frac{2}{5} \div \frac{3}{4} = ?
\]
\[
\frac{2}{5} \div \frac{3}{4} = ?
\]
\[
\frac{2}{5} \div \frac{3}{4} = ?
\]
\[
\frac{2}{5} \div \frac{3}{4} = ?
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\[
\frac{2}{5} \div \frac{3}{4} = ?
\]
\[
\frac{2}{5} \div \frac{3}{4} = ?
\]
\[
\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3} = \frac{2 \times 4}{5 \times 3} = \frac{8}{15}
\]
\[
\frac{2}{5} \div \frac{3}{4} = \left( \frac{2}{5} \div 3 \right) \times 4 = \frac{2}{5 \times 3} \times 4 = \frac{2 \times 4}{5 \times 3}
\]

\[
= \frac{2 \times 4}{5 \times 3} = \frac{\Delta}{O} \times \frac{\#}{*}
\]
\[
\frac{2}{5} \div \frac{3}{4} = \left(\frac{2}{5} \div 3\right) \times 4 = \frac{2}{5 \times 3} \times 4 = \frac{2 \times 4}{5 \times 3}
\]

\[
\frac{2 \times 4}{5 \times 3} = \frac{2}{5} \times \frac{4}{3}
\]

\[
\frac{2}{5} \div \frac{3}{4} = \frac{2 \times 4}{5 \times 3}
\]
What fraction can you multiply $\frac{2}{5}$ by to get the same calculation as above?

When you divide by a fraction, invert the denominator and the numerator of the divisor and then multiply the dividend by the resulting new fraction.
Properties of Division

• When the divisor = 1, the quotient is the same as the dividend. [“any number divided by 1 equals the number itself”]

• When the dividend and divisor are both multiplied or divided by the same number, the quotient does not change. M4N4(d)

Example:

$$15 \div 3 = 30 \div 6 = 5 \div 1$$
So, how can we apply these properties to the following?

\[
\frac{9}{14} \div \frac{3}{4} = ?
\]
To change $\frac{3}{4}$ into 1

$\frac{3}{4} \times \frac{4}{3} = 1$
Therefore,

\[
\frac{9}{14} \div \frac{3}{4} = \left( \frac{9}{14} \times \frac{4}{3} \right) \div \left( \frac{3}{4} \times \frac{4}{3} \right)
\]

\[
\left( \frac{9}{14} \times \frac{4}{3} \right) \div 1 = \frac{9}{14} \times \frac{4}{3}
\]
Reference


The series is available through Global Education Resources

www.globaledresources.com