Kyozaikenkyu: A key to conduct lesson study effectively

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What is *kyozaikenkyu*?

教材研究
What is kyozaik kenkyu?
What is kyozaï kemkyu?

教材 研究
What is *kyozai kenkyu*?

*kenkyu*: study or research
What is kyozai kenkyu?

教 材研究

“kyo”: to teach or instruct
What is *kyozai kenkyu*?

教材研究

“*zai*”: raw materials
Kyozai is an actualization of educational content and includes learning goals. It is important that kyozai and subject matter content (specific knowledge and procedures to be learned through lessons) are distinguished. It is possible to explore the same subject matter with different kyozai, or we can investigate different subject matter with the same kyozai. …
... To distinguish *kyozai* and subject matter content means to clearly articulate what specific subject matter content will be (can be) taught through a particular *kyozai*. If this relationship is not clearly articulated, the selection of *kyozai* or its interpretation becomes unclear. That, in turn, may make the purpose of the lesson unclear.

(Yokosuka, 1990)
...the entire process of research activities related to *kyozai*, beginning with the selection/development, deepening the understanding of the true nature of a particular *kyozai*, planning a lesson with a particular *kyozai* that matches the current state of the students, culminating in the development of an instructional plan.

(Yokosuka, 1990)
Two types
  ◦ study of existing kyozai
  ◦ study to develop new kyozai

Two stages
  ◦ deeply examining kyozai to fully understand its nature and purpose
  ◦ plan a lesson using a particular kyozai

Two perspectives
  ◦ from teachers’ perspectives
  ◦ from children’s perspectives
Textbooks are collections of *kyozai*, i.e., raw materials for instruction.

*Kyozaikenkyu* of textbooks is an important step to process the raw materials.
- Investigate textbooks from other grades.
- Investigate different textbooks.
- Solve all problems.

Textbooks are only the “entrance” to *kyozaikenkyu*.
- Investigate the standards
- Investigate other instructional resources
- Investigate existing research/practitioners’ wisdom
- etc.
What does this idea really mean? What do we want students to understand?

How does this idea relate to other ideas?

What ideas do students already understand that can be used as a starting point for this new idea?

Why is this particular problem useful in helping students develop this new idea?

How can students solve this problem using what they already know, and how can their solution strategies be used to develop this new idea?

What are common mistakes? Why do students make such mistakes? How should teachers respond to those mistakes?

What new ideas are students expected to build using this idea in the future?

etc.
Understand Mathematics

Understand Scope & Sequence

Understand Children’s Mathematics

Explore Possible Problems, Activities and Manipulatives

Instruction Plan
Let’s conduct kyozaikenkyu

- Grade 5: Multiplication by fractions

\[ \frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \]
All the pans of brownies are square. A pan of brownies costs $12. You can buy any fractional part of a pan of brownies and pay that fraction of $12. For example, \( \frac{1}{2} \) of a pan costs \( \frac{1}{2} \) of $12.

1. Mr. Williams asks to buy \( \frac{1}{2} \) of a pan that is \( \frac{2}{3} \) full. What fraction of a whole pan does Mr. Williams buy? What does he pay?
教材研究: Let’s solve the problem
1. Mr. Williams asks to buy \( \frac{1}{2} \) of a pan that is \( \frac{2}{3} \) full. What fraction of a whole pan does Mr. Williams buy? What does he pay?

What fraction of a whole pan?
\[
\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}
\]

What does he pay?
\[
\frac{2}{6} \times 12 = 4
\]
What is the difference between “multiplying a fraction by a whole number” (Grade 4) and “multiplying a fraction or whole number by a fraction” (Grade 5)?

Which grade should this calculation be studied?

Why do we distinguish them?
MAFS.3.OA.1.1

- Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as $5 \times 7$.

- How do we read “$5 \times 7$”? 
  - 5 multiplied by 7?
  - 7 multiplied by 5?
MAFS.4.NF.1.1

Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

CCSS Progression: “Grade 4 students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction.”
5 × 7: “7 (is) multiplied by 5”

- (number of groups) × (group size)
- “(group size) is multiplied by (number of groups)”

\[
\frac{2}{6} \times 12 = 4 \quad \Rightarrow \quad \text{multiplying a whole number by a fraction (Grade 5)}
\]

\[
12 \times \frac{2}{6} = 4 \quad \Rightarrow \quad \text{multiplying a fraction by a whole number (Grade 4)}
\]
12 × \( \frac{2}{6} \) can be interpreted as 12 groups of \( \frac{2}{6} \).

\( \frac{2}{6} \times 12 \) means “\( \frac{2}{6} \) groups of 12” – what does “\( \frac{2}{6} \) groups” mean?

In Grade 5, we must help students expand their understanding of the meaning of multiplication.
What do we want students to understand?
MAFS.5.NF.2.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

a. Interpret the product \((a/b) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = (ac/bd)\).

b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.
Interpret multiplication as scaling (resizing), by:

a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying $a/b$ by 1.
What do we want students to understand?

- Apply and extend their understanding of multiplication.
- Interpret the product \((a/b) \times q\)
  - as \(a\) parts of a partition of \(q\) into \(b\) equal parts
  - as the result of a sequence of operations \(a \times q \div b\)
  - create a story context for this equation
- Apply their understanding of multiplication of fractions to area of rectangles.
- Interpret the effects of multiplication without performing the calculation.
- Explain
  - If multiplier > 1 \(\rightarrow\) product > multiplicand
  - If multiplier < 1 \(\rightarrow\) product < multiplicand
Given these goal understandings, how appropriate is the textbook task?
All the pans of brownies are square. A pan of brownies costs $12. You can buy any fractional part of a pan of brownies and pay that fraction of $12. For example, \( \frac{1}{2} \) of a pan costs \( \frac{1}{2} \) of $12.

1. Mr. Williams asks to buy \( \frac{1}{2} \) of a pan that is \( \frac{2}{3} \) full. What fraction of a whole pan does Mr. Williams buy? What does he pay?
Teacher Guide

Goals

- Estimate products of fractions
- Use models to represent the product of two fractions
- Understand that finding a fraction of a number means multiplication

“... allow students to find ways to make sense of the problem using the models... think about what it means to find a ‘part of a part.’ Understanding that ‘part of a part’ means ‘×’ is raised in Question 3.”
But,

- How would students know that they are finding a product if they don’t know what they are doing is multiplication first?
- How would students understand “part of a part” means “×”?
- How does this problem connect to the standard, “Interpret the product \((a/b) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\)”?
- …
How can students develop the goal understanding?

- What do students already know that they can use to develop these goal understanding and how?
- How do students need to understand this particular idea?
## Students’ Prior Experiences

<table>
<thead>
<tr>
<th>Grade</th>
<th>Content: Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 2</td>
<td>• Partitioning geometric figures: “use fraction language to describe partitions of shapes into equal shares”</td>
</tr>
</tbody>
</table>
| 3     | • Fractions as numbers  
|       |   • Understand unit fractions as building blocks  
|       |   • Fractions on number line  
|       |   • Equivalent fractions (awareness, in simple cases) |
| 4     | • Creating equivalent fractions  
|       | • Adding and subtracting fractions (like denominators)  
|       | • Multiplication  
|       |   • A non-unit fraction as a product of a whole number and a unit fraction  
|       |   • Multiplying a fraction by a whole number |
| 5     | • Adding and subtracting fractions (unlike denominators)  
|       | • Fractions as quotients  
|       | • Dividing a whole number by a unit fraction and dividing a unit fraction by a whole number |
## Students’ Prior Experiences

<table>
<thead>
<tr>
<th>Grade</th>
<th>Content: Multiplication</th>
</tr>
</thead>
</table>
| 3     | • Introduction of multiplication: equal groups  
       | • Introduction of division: sharing and measurement  
       | • Connecting multiplication to area |
| 4     | • Multiplication as comparison: N times as much  
       | • Multiply larger numbers: using strategies based on place value and the properties of operations  
       | • Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. |
| 5     | • Fluently multiply with the standard algorithm |
How is this topic treated in various curriculum materials?
Let’s Think about How to Multiply by Fractions

With 1 dL of paint, we can paint \(\frac{4}{5}\) m\(^2\) of boards.

How many m\(^2\) of boards can we paint with \(\frac{2}{3}\) dL of this paint?

\[
\frac{4}{5} \times \frac{2}{3} = \frac{8}{5} \quad \text{Answer:} \quad \frac{8}{5} \text{ m}^2
\]

Let’s think about what math sentence we should write.

**Math Sentence**

- Shinji: The amount of boards we can paint with \(\frac{2}{3}\) dL will be 2 times the area we can paint with 1 dL. So, with \(\frac{2}{3}\) dL...
- Kaori: Explain the reason for your math sentence.

Area we can paint with 1 dL \(\times\) Amount of paint (dL) = Area we can paint
Even when the amount of paint used is a fraction, we can use multiplication to calculate the total area that can be painted, just like we did with whole numbers.

\[
\frac{4}{5} \times \frac{2}{3}
\]

Let’s think about how to calculate.

First, we can find the area of boards we can paint with \(\frac{1}{3}\) dl, then...

I wonder if we can change \(\frac{2}{3}\) into a whole number just as we did with decimal multipliers...

Compare the last part of the math sentences in these two students’ ideas.
This textbook tries to establish the meaning of multiplication first.
Mathematics International (Tokyo Shoseki, Grade 5)
Strengths

- This textbook tries to establish the meaning of multiplication first.
- This textbook seems to make connection to “Interpret the product \((a/b) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\)” much more explicitly.
Makayla said, "I can represent $3 \times \frac{2}{3}$ with 3 rectangles each of length $\frac{2}{3}$.

Connor said, “I know that $\frac{2}{3} \times 3$ can be thought of as $\frac{2}{3}$ of 3. Is 3 copies of $\frac{2}{3}$ the same as $\frac{2}{3}$ of 3?”

1. Draw a diagram to represent $\frac{2}{3}$ of 3.
2. Explain why your picture and Makayla’s picture together show that $3 \times \frac{2}{3} = \frac{2}{3} \times 3$.
3. What property of multiplication do these pictures illustrate?
The purpose of this task is to

a. have students think about the meaning of multiplying a number by a fraction, and

b. use this burgeoning understanding of fraction multiplication to make sense of the commutative property of multiplication in the case of fractions.

In principle, students have already had a chance to see why multiplication of whole numbers is commutative by looking at arrays or rectangles in third grade. After seeing the kinds of pictures used in this task and its solution, students should also look at rectangles with fractional side lengths to help reinforce their understanding of the commutative property of multiplication.

https://www.illustrativemathematics.org/illustrations/321
\( \frac{2}{3} \) of 3 \( m = 3 \) pieces of \( \frac{2}{3} \) \( m \)
The purpose of this task is to

a. have students think about the meaning of multiplying a number by a fraction, and

b. use this burgeoning understanding of fraction multiplication to make sense of the commutative property of multiplication in the case of fractions.

In principle, students have already had a chance to see why multiplication of whole numbers is commutative by looking at arrays or rectangles in third grade. After seeing the kinds of pictures used in this task and its solution, students should also look at rectangles with fractional side lengths to help reinforce their understanding of the commutative property of multiplication.

Students should have some prior experience with interpreting $\frac{a}{b} \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts before they are ready to tackle parts (b) and (c) of the task.
How can we teach this particular topic effectively?

- How should we sequence the goal understandings?
- What are common misunderstandings children have about this topic?
- What tools/manipulatives can students use to develop their understanding?
Possible sequence of ideas

- Developing the meaning of multiplying by fractions
- Thinking about ways to find the product
- Developing computational fluency
- Examining the relationship between the product and the multiplicand based on the size of the multiplier
Developing the meaning

- Making connection to what students already know
  - Use the same problem contexts to discuss multiplying fraction by a whole number and multiplying by a fraction
  - Use representations – such as double number line or area
  - Use equation with words
Important Prerequisite Understanding

- Understanding the meaning of multiplication
  - Equal groups
  - Comparison

- Familiarity with representations
  - Area model of multiplication
  - Double number line

- Reading and writing expressions and equations

- etc.
Where does the research lesson fit in the sequence?
What will be the main task?
How should we pose the task?
How will students solve the task?
How can we make use of students’ solutions to engage them in rich mathematical discussion?
How can we assess students’ understanding?
How can we summarize the lesson?
etc.
Identify the goal understandings.
  ◦ Study the textbooks, standards, and other curriculum resources.

Identify the potential difficulties.
  ◦ Study published reports.
  ◦ Examine own experiences teaching the topic.

Develop a hypothetical learning path.
  ◦ Solve all possible problems and examine various solution methods.
  ◦ Explore various tools/manipulatives.

Develop a lesson plan for a particular lesson.
教材研究 and Lesson Plan

- *Kyozaikenkyu* is the basis of the research lesson.
- Observers need to know
  - *Why a specific task was chosen*
  - *Why a task is posed in a particular way*
  - *Why a particular tool is provided (or not provided)*
  - *etc.*
- Your *kyozaikenkyu* must serve as the starting point for another group
Japanese saying

To teach one, you need to learn ten.

After you learn ten, you must discard nine.
Thank you!