What Do You Mean, 50% is 0.5? It was 20 Earlier!

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http://science.kennesaw.edu/~twatanab
Place the following numbers on the number line below.

a) $\frac{2}{3}$  

b) 1.4  

c) 80%
Percents in Standards

• … (students) work flexibly with fractions, decimals, and percents to solve problems (PSSM, Grades 6-8)
Percents in GA Standards

• Students will understand the meaning of percentage. (Grade 5)
  a. Model percent on 10 by 10 grids.
  b. Apply percentage to circle graphs.

• Use fractions, decimals, and percents interchangeably. (Grade 6)
Percents in VA Standards

• The student will identify representations of a given percent and describe orally and in writing the equivalence relationship between fractions, decimals, and percents. (SoL: 6.1)

• The student will compare, order, and determine equivalent relationships between fractions, decimals, and percents, including use of scientific notation for numbers greater than 10. (SoL: 7.1)
From VA Math Framework

Teacher Notes

- *Percent* means “per 100” or how many “out of 100”; *percent* is another name for *hundredths*.
- A number followed by a percent symbol (%) is equivalent to that number with a denominator of 100.
- Percents can be expressed as decimals.
- A fraction can be rewritten as an equivalent fraction with a denominator of 100, and, thus, as a decimal or percent.
From VA Math Framework

Teacher Notes (cont.)

- Decimals, fractions, and percents can be represented using concrete materials (e.g., base-10 blocks, decimal squares, or grid paper).

- Percents should be represented by drawing a shaded region on a 10-by-10 grid to represent a given percent.

- Percents are used in real life for taxes, sales, data description, and data comparison.
From VA Math Framework

Essential Understanding

All students should:

• understand that percent is a way of representing fractions and decimals.

• understand that a number can be written as a fraction, decimal, or percent.

• understand that percent is a method of standardization that is efficient because each number is always based on 100ths.

• understand that percents are used in real-life applications to compare or describe data.
Percents on a $10 \times 10$ Grid

- Understand that percent means how many out of 100.
Percents on a 10 x 10 Grid

• Understand that percent means how many out of 100.

This shows 40 %
Percents on a 10 x 10 Grid

- Understand that percent means how many out of 100.

This shows 40 %

But it is also 4/10 and 0.4

Understand that a number can be written as a fraction, decimal or percent.
Are percents numbers?

• Understand that percent is a way of representing fractions and decimals.
• Understand that a number can be written as a fraction, decimal, or percent.
If percents were numbers...

“50% gallon container of milk”

“75% mile to next exit”

50% + 75% = 125%

Percents may not really be numbers.
How are percents introduced in Japan?

Grade 5 classes are participating in a city-wide basketball league. The table below shows their current records. Who has the best record?

<table>
<thead>
<tr>
<th>Team</th>
<th>Games</th>
<th>Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>Yellow</td>
<td>15</td>
<td>6</td>
</tr>
<tr>
<td>Blue</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Green</td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>
It’s easy to compare when...

• Both teams have won the same number of games
  – The team with a fewer games has a better record

• Both teams have played the same number of games
  – The team with more wins has a better record
What to do otherwise...

<table>
<thead>
<tr>
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<tbody>
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</tr>
</tbody>
</table>

• Think about the portion (fraction) of the games that each team won
• Make the number of games (whole) the same
Similar ideas: rates

• Which car was faster?

<table>
<thead>
<tr>
<th>Cars</th>
<th>Distance</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>80 miles</td>
<td>90 min.</td>
</tr>
<tr>
<td>B</td>
<td>70 miles</td>
<td>90 min.</td>
</tr>
<tr>
<td>C</td>
<td>80 miles</td>
<td>100 min.</td>
</tr>
</tbody>
</table>

Compare the distance traveled in the same amount of time
or

Compare the time it took to travel the same distance
Similar ideas: rates

- Which car was faster?

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<tr>
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</table>

When comparing several cases, it is easier if we make the time (or the distance) all the same.

Use one unit time - speed in miles/min.
What fraction of the games did each team win?

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• 7/10 and 9/15 are the “number” of wins for each game these teams played - wins per game.
• We made the number of games = 1.
Making the number of games the same

• Which team has the best record?

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</tr>
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<tbody>
<tr>
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<td>20</td>
<td>11</td>
</tr>
<tr>
<td>B</td>
<td>40</td>
<td>23</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>25</td>
<td>13</td>
</tr>
</tbody>
</table>
Making the number of games the same

- Which team has the best record?

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<th>Wins</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
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Making the number of games the same

• Which team has the best record?

<table>
<thead>
<tr>
<th>Team</th>
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<th>Wins</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>100</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
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<td>100</td>
<td>40</td>
<td>24</td>
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Making the number of games the same

- Which team has the best record?

<table>
<thead>
<tr>
<th>Team</th>
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<th>Wins</th>
<th>percent</th>
</tr>
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</table>
From VA Math Framework

Essential Understanding

• understand that percent is a method of standardization that is efficient because each number is always based on 100ths.
If 60% = 6/10, isn’t percent a number?

• No.
• If a team wins 6 out of 10 games (60 %) in the first half of a season and 7 out of 10 games (70%) in the second half of the season, did it win 60 + 70 = 130% of the season?

• Or, \[
\frac{6}{10} + \frac{7}{10} = \frac{13}{20}
\]?
If 60% = 6/10, isn’t percent a number?

• Percents tell us relative quantities.
• Fractions and decimals may be used to express relative quantities - i.e., by considering the base of comparison = 1
• Only when we are talking about relative quantities, we may use fractions, decimals, and percents interchangeably.
Implications to fraction teaching

• When we introduce fractions as a part of a whole, we may be talking about a relative quantity.

• We can perform arithmetic operations only with numbers - we must help students develop fractions as numbers before we expect them to make sense of fraction arithmetic.
In Nishikawa City, each person produced 800g of waste a day last year. This year they are campaigning for each person to reduce their waste by 120g a day. If you set last year's amount as the base quantity, what is the relative value of the amount of reduction?
In Nishikawa City, each person uses 225kg of paper every year on average. 52% of the paper is recycled.

How many kilograms of paper will be recycled out of 225kg?

52% of 225kg is 0.52 times as much as 225kg.

\[225 \times 0.52 = \text{kg}\]
In Nishikawa City, each person produced 800g of waste each day last year. This is 125% of the amount produced 10 years ago.

What was the amount of waste produced 10 years ago?

1. Make the amount 10 years ago $x$g and write a multiplication sentence. Then find $x$.

$$x \times 1.25 = 800$$

$$x = 800 \div 1.25$$

Answer: $x$g
Naoko's school collects aluminum cans. Last month they collected 350 cans. This month they collected 18% more than last month. How many aluminum cans were collected this month?
This month: □ cans
Last month: 350 cans

\[ 350 + 350 \times 0.18 \]
Answer: □ cans

\[ 350 \times (1 + 0.18) \]
Answer: □ cans

Ritsuko

Minoru
12 Percentage and Graphs

On the Internet, there are many websites about waste management issues.

We asked a city official to find out how much waste is produced by 3 elementary schools in Nishikawa City each day.

<table>
<thead>
<tr>
<th>School</th>
<th>Total amount of waste</th>
<th>Amount of paper</th>
</tr>
</thead>
<tbody>
<tr>
<td>East Elementary</td>
<td>80 kg</td>
<td>36 kg</td>
</tr>
<tr>
<td>West Elementary</td>
<td>70 kg</td>
<td>35 kg</td>
</tr>
<tr>
<td>North Elementary</td>
<td>60 kg</td>
<td>33 kg</td>
</tr>
</tbody>
</table>

East Elementary School produces the most paper waste. East Elementary School produces the most waste, too.

Let’s think about how we can represent the relationship of the parts and the whole!

1 Relative Value and Percentage

1 Among East, West and North elementary schools, which school has more paper waste in comparison to the total waste?
Let’s think about how to compare them!

1 At which school was the amount of paper waste more than \( \frac{1}{2} \) of the total amount of waste?
The graph below shows the relative values of different kinds of waste from Nishikawa City. Let's find the relative value of each kind based on the total amount of waste!

Relative Values of Kinds of Waste in Nishikawa City

<table>
<thead>
<tr>
<th>Paper</th>
<th>Kitchen Garbage</th>
<th>Plastic</th>
<th>Wood</th>
<th>Other</th>
</tr>
</thead>
</table>

0 10 20 30 40 50 60 70 80 90 100 %
Percents are not a focal point, but...

• they are a part of a network of multiplicative concepts
• they should be developed carefully
  – they are not another representation of rational numbers
  – fractions and decimals may be used in place of percents BECAUSE they may be used to represent relative values
  – percents should not be used in place of fractions and decimals as numbers